

Implementation of the Quantum Fourier Transform algorithm in qSOA[®]

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1 Quantum Fourier Transform

1.1 The problem

The Fourier Transform (FT) is one of the most useful mathematical tools in modern science and engineering. The Quantum Fourier Transform (QFT) is the quantum analog of the discrete Fourier transform (DFT). It is used as a basis shifting tool between the canonical computational basis and the Fourier basis. It allows access to hidden patterns and information stored in the phases and relative magnitudes of a circuit and, in many cases, to perform computations in a more efficient and elegant way. It is a very useful and fundamental subroutine in many quantum algorithms, such as Shor's algorithm.

1.2 Classical version

The discrete Fourier transform acts on a vector $(x_0, \dots, x_{N-1}) \in \mathbb{C}^N$ and maps it to the vector $(y_0, \dots, y_{N-1}) \in \mathbb{C}^N$ according to the general mathematical formula:

$$y_k = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} x_j e^{2\pi i \frac{jk}{N}}$$

1.3 Quantum version

We can rewrite the general formula into a quantum form, replacing vectors x and y with a quantum state $|j\rangle$ as input and denoting the output as $|k\rangle$. We also assume $N = 2^n$, where n is the number of qubits.

$$QFT |j\rangle = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} e^{\frac{2\pi i j k}{N}} |k\rangle \quad (1)$$

On the other hand, we can express arbitrary n qubits in the quantum state Ψ using the following generic formula (where $N = 2^n$).

$$|\Psi\rangle = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} x_j |j\rangle \quad (2)$$

Therefore, we can combine both formulas (1) and (2) to obtain a generalized definition for the QFT acting on an arbitrary n qubit quantum state:

$$QFT |\Psi\rangle = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} \sum_{k=0}^{N-1} x_k e^{2\pi i j k / N} |j\rangle \quad (3)$$

where $|\Psi\rangle = |x_n x_{n-1} \dots x_1 x_0\rangle$ and $R_k = P\left(\frac{\pi}{2^{k-1}}\right) = \begin{pmatrix} 1 & 0 \\ 0 & e^{\frac{i\pi}{2^{k-1}}} \end{pmatrix}$. The generic QFT circuit has the form shown in the next figure:

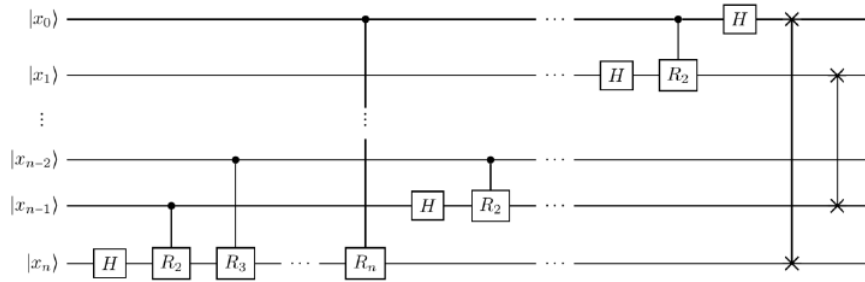


Figure 1: QFT circuit of n-qubits

Since $QFT |0\rangle^{\otimes n} = H^{\otimes n} |0\rangle^{\otimes n}$, in this tutorial we are going to implement the QFT algorithm on the 5-qubit circuit state $|00000\rangle$ and then apply a Hadamard gate on each qubit. Now, the expected value after measuring is $|00000\rangle$ with probability 1. Therefore, the circuit we are going to implement is:

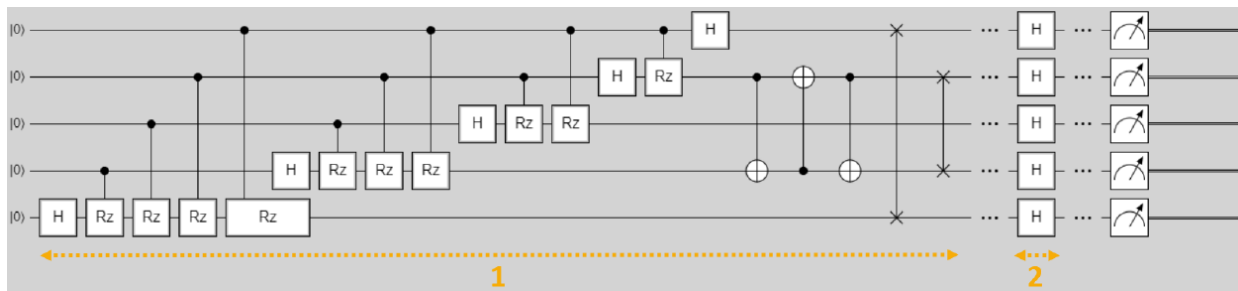


Figure 2: QFT algorithm in QPath[®]

1.4 Usefulness of the algorithm

QFT can compute the Fourier transform of an n-qubit register with a time complexity of $\mathcal{O}(n^2)$, being more efficient than its classical counterpart, which has complexity of $\mathcal{O}(n2^n)$. However, if we measure the entire output state, then the complexity of the QFT becomes $\mathcal{O}(n^22^n)$, thus losing its apparent advantage, indicating that the quantum enhancement of the Fourier transform is fully exploited for algorithms where not all qubits in the circuit are required to be measured.

2 Implementation of the algorithm in qSOA[®]

Once we are aware of how the algorithm works, we are ready to implement it in qSOA[®]. This will allow us to create the circuit and execute it in different quantum computing providers, among many other things.

The process of implementing an algorithm in qSOA[®] is comprised of four steps:

1. Setting up qSOA[®] and selecting the quantum solution
2. Creating a circuit with the algorithm and assigning it to the solution

3. Introducing a circuit flow to control the number of launches of the algorithm
4. Executing the flow on different quantum devices.

As can be seen in qSOA's[®] manual, ref.[1], there are multiple ways to secure the connection depending on the context. Following the best practices of qSOA[®], in this tutorial we are integrating the security in the code. Similarly, one can work with asynchronous or synchronous programming. Keeping in mind the purpose of this tutorial we will use the synchronous version.

2.1 Setting up qSOA[®]

Firstly, we import the SDK that has been previously installed, see ref.[1], and create the qSOA[®] workspace to work with.

```
[1]: from QuantumPathQSOAPySDK import QSOAPlatform # Import SDK
```

```
[2]: # Create qSOA workspace, login manually
qsoa = QSOAPlatform()

username = 'username'
password = 'password' # password encrypted in SHA-256

authenticated = qsoa.authenticateEx(username, password)

print('Authentication completed:', authenticated)
```

Authentication completed: True

Then we review the existing solutions and select the one we are interested in.

```
[3]: # Get catalogs
solutionList = qsoa.getQuantumSolutionList()
print(" ", solutionList)
idSolution = int(input("Select idSolution: "))
```

```
{'12345': 'QS_GateTutorials'}
Select idSolution: 12345
```

2.1.1 Securing the connection

As has been said, qSOA[®] allows multiple business development contexts. Therefore, the user can secure the connection through a configuration file, *.qpath*, useful at a personal level, or in parameterized way, as it is done here.

2.2 Assigning a circuit to the solution

Once the solution has been selected, we must link it to the circuit that we are interested in implementing. Therefore, we first need to define the circuit and create it.

2.2.1 Defining the circuit

As we are working with a gates circuit, we can either enter the circuit in *visual language*, VL, or *intermediate language*, IL. In this case we are going to work with VL and, therefore, we can either enter the circuit as a *String* or as a *CircuitGates* object, which is what we are going to go for.

In order to define the circuit shown in figure 2, we just need to write the gates we want to use, in the correspondent qubits.

```
[4]: ## Defining circuit w/ Circuit Gates
n = 5 # Number of qubits in the circuit

circuitG = qsoa.CircuitGates()
for j in range(n-1,-1,-1):
    circuitG.h(j)
    for i in range(j-1,-1,-1):
        circuitG.mcg(i, circuitG.rz(j, 'pi/4', False))
circuitG.cx(1,3)
circuitG.cx(3,1)
circuitG.cx(1,3)
circuitG.swap(0,4)
circuitG.swap(1,3)
circuitG.barrier()
circuitG.h()
circuitG.barrier()
circuitG.measure()

print(circuitG)
# print(circuitG.getCircuitBody())
```

```
<QuantumPathQSOAPySDK.classes.CircuitGates.CircuitGates object at
0x0000016DCA49CBE0>
```

Note: Every gate can be applied to a single qubit, a list of them, introduced as a list, or to every qubit in the circuit, using ().

2.2.2 Creating the circuit

In order to create the circuit we are going to use the *createAssetSync* function. This function receives the following fields as inputs:

- *idSolution*: to associate the circuit with the solution we have selected before.
- *assetName*: to set the name of the circuit.
- *assetNamespace*: to associate the circuit with a class of circuits that share something in common. In this case, we associate the circuit with a set of basic circuits.
- *assetDescription*: to write a brief description of the algorithm.

- `assetBody`: to select the circuit we have previously defined.
- `assetType`: to select if we are working with a gates circuit or an annealing one.
- `assetLevel`: to select either visual language or intermediate language, according to the definition of the circuit.

```
[5]: ## Circuit creation
assetName = 'QC_qSOA_QFT'
assetNamespace = 'Manual.Gates.QFT'
assetDescription = 'Creating the QFT circuit from qSOA'

assetBody = circuitG
assetType = 'GATES'
assetLevel = 'VL'

CircuitManagementResult = qsoa.createAssetSync(idSolution, assetName,
assetNamespace, assetDescription, assetBody, assetType, assetLevel)
```

Note: Note that the synchronous version of this function is being used. This is because the *createAsset* function creates, compiles and transpiles the asset, and the synchronous function waits for all of it to be done before moving on. For a better understanding of how this functions works, we recommend using the asynchronous version, *createAsset*, and the *getAssetManagementResult* function.

2.3 Assigning a circuit flow to the circuit

The quantum flow is a box diagram that will allow us to control the number of runs of our algorithm. This is very useful, as the results obtained in quantum computing have a certain probability associated with them. That means that the more times we run the algorithm, the more robust the results will be.

2.3.1 Defining the flow

As happens with the circuit, the flow can be written in VL or IL. If we choose to do it with VL then it can be entered as a *String* or a *CircuitFlow* object. Otherwise, it can only be entered as a *String*. For this example we are choosing VL and *CircuitFlow*.

In order to define a flow we need:

1. Starting node
2. Initializing node: usually set to 0
3. Circuit node: where we write the circuit we want to launch
4. Repeat node: where the number of repetitions can be established
5. End node

6. Links between each node we have created

```
[8]: # Defining flow w/ CircuitFlow
flow = qsoa.CircuitFlow()
startNode = flow.startNode()
initNode = flow.initNode(0)
circuitNode = flow.circuitNode('Manual.Gates.QFT.QC_qSOA_QFT')
# Namespace + CircuitName
repeatNode = flow.repeatNode(1000)
endNode = flow.endNode()

flow.linkNodes(startNode, initNode)
flow.linkNodes(initNode, circuitNode)
flow.linkNodes(circuitNode, repeatNode)
flow.linkNodes(repeatNode, endNode)
print(flow.getFlowBody())

[8]: {'class': 'go.GraphLinksModel', 'nodeDataArray': [{'category': 'Start', 'text':
'Start', 'key': -1, 'loc': ''}, {'category': 'Init', 'text': '0', 'key': -2,
'loc': ''}, {'category': 'Circuit', 'text': 'Manual.Gates.QFT.QC_qSOA_QFT', 'key':
→ -3, 'loc': ''}, {'category': 'Repeat', 'text':
'1000', 'key': -4, 'loc': ''}, {'category': 'End', 'text': 'End', 'key': -5,
'loc': ''}], 'linkDataArray': [{'from': -1, 'to': -2, 'points': []}, {'from':
-2, 'to': -3, 'points': []}, {'from': -3, 'to': -4, 'points': []}, {'from': -4,
'to': -5, 'points': []}]}
```

2.3.2 Creating the flow

In order to create the flow we are using *createAssetFlowSync*, although the *createAssetSync* function would also work. The inputs that this function requires are:

- *idSolution*: to associate the flow with the solution we have selected before.
- *assetName*: to set the name of the flow.
- *assetNamespace*: to associate the flow with a class of flows that share something in common. In this case, we associate the flow with a set of basic flows.
- *assetDescription*: to write a brief description of the algorithm.
- *assetBody*: to select the flow we have previously defined.
- *assetLevel*: to select either VL or IL, according to the definition of the flow.
- *publish*: to select if we want to publish the flow on qSOA® or not.

```
[9]: ## Flow creation
assetName = 'QF_qSOA_QFT'
assetNamespace = 'Manual.Gates.QFT'
```



```

assetDescription = 'Creating the QFT flow from qSOA'
assetPublication = True

assetBody = flow
assetType = 'FLOW'
assetLevel = 'VL'

FlowManagementResult = qsoa.createAssetFlowSync(idSolution, assetName,
assetNamespace, assetDescription, assetBody, assetLevel,assetPublication)

```

Note: Note that the synchronous version of this function is being used. This is because the *createAssetFlow* function creates, compiles and transpiles the flow, and the synchronous function waits for all of it to be done before moving on. For a better understanding of how this functions works, we recommend using the asynchronous version, *createAssetFlow*, and the *getAssetManagementResult* function.

2.4 Implementation of the algorithm

Note: If you would like to execute any other previously created flow, you can do so by implementing this part of the tutorial. You will need to specify the ID Solution and the ID of the flow.

2.4.1 Executing the algorithm

We are now ready to execute the algorithm, so lets see on what platforms we can do so, and select the ones we are interested in.

```

[12]: deviceList = qsoa.getQuantumDeviceList(idSolution)
print('Device List:', deviceList)
DeviceID = input("Select a device to run the flow in: ")

```

```

Device List: {'14': 'AMAZON BRAKET 25qubits Local Simulator', '2': 'QISKIT Local Simulator', '1': 'Q# Local Simulator Framework'}
Select a device to run the flow in: 14

```

Now, we proceed to run the quantum algorithm with the *runQuantumApplicationSync* function.

```

[13]: exe_application = qsoa.runQuantumApplicationSync('NameTheTask', idSolution,
→FlowID, DeviceID)

```

Note: Note that the synchronous version of this function is being used. This is because *runQuantumApplication* launches an execution, and the synchronous function waits for the execution to be done before moving on. If we do not know how long the execution is going to take (due to the execution itself or the queue), or if it takes too long, the asynchronous version presents a clear advantage. However, for a better understanding of how this functions work, we recommend using the asynchronous version, *runQuantumApplication*.

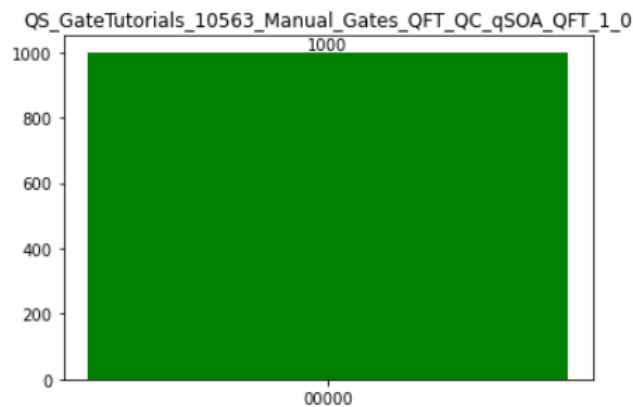
The function `runQuantumApplicationSync` gives an application object as output. For us to manage the results we need the `getQuantumExecutionResponse` function that returns the results as an execution object.

```
[14]: restok_execution = qsoa.getQuantumExecutionResponse(exe_application.
        ↳getExecutionToken(), idSolution, FlowID)
restok_histogram = restok_execution.getHistogram()
print(restok_histogram)
```

```
{'QS_GateTutorials_12345_Manual_Gates_QFT_QC_qSOA_QFT_1_0': {'00000': 1000}}
```

Now that we have the results we asked for, we can proceed to represent them with the function `representResults`.

```
[15]: # Circuit gate representation
gates_representation = qsoa.representResults(restok_execution)
```



2.4.2 Multiple devices

If we want to run the algorithm in multiple devices at the same time, we can do so by creating an array with the information needed and proceeding the same way as before.

```
[16]: deviceList = qsoa.getQuantumDeviceList(idSolution)
print('Device List:', deviceList)
```

```
Device List: {'14': 'AMAZON BRAKET 25qbits Local Simulator', '2': 'QISKIT Local Simulator', '1': 'Q# Local Simulator Framework'}
```

```
[17]: ## Run Quantum Gates Application
exe_ApplicationNames = ['Task_Amazonsim', 'Task_Qiskitsim', 'Task_QSharpsim']
exe_IdDevices = [14,2,1]

exe_Applications = [0] * len(exe_ApplicationNames)
```

```
for i in range(len(exe_ApplicationNames)):
    exe_Applications[i] = qsoa.runQuantumApplicationSync(exe_ApplicationNames[i],
        idSolution, FlowID, exe_IdDevices[i])
```

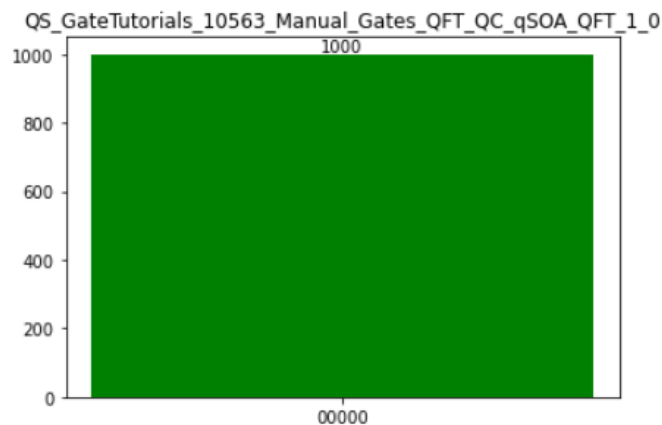
```
[18]: # Get quantum execution response with execution token
restok_Executions = [0] * len(exe_ApplicationNames)
restok_Histograms = [0] * len(exe_ApplicationNames)

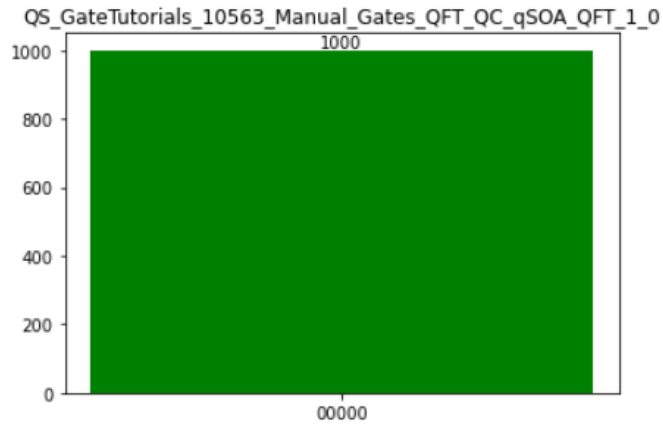
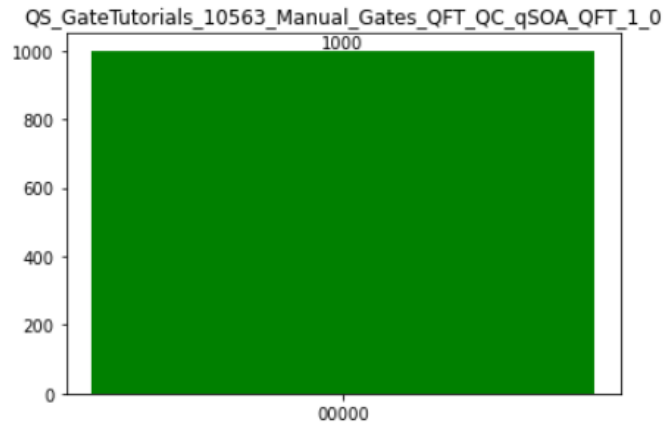
for i in range(len(exe_ApplicationNames)):
    restok_Executions[i] = qsoa.getQuantumExecutionResponse(exe_Applications[i].
        →getExecutionToken(), idSolution, FlowID)
    restok_Histograms[i] = restok_Executions[i].getHistogram()

print(restok_Histograms)
```

```
[{'QS_GateTutorials_12345_Manual_Gates_QFT_QC_qSOA_QFT_1_0': {'00000': 1000}},
{'QS_GateTutorials_12345_Manual_Gates_QFT_QC_qSOA_QFT_1_0': {'00000': 1000}},
{'QS_GateTutorials_12345_Manual_Gates_QFT_QC_qSOA_QFT_1_0': {'00000': 1000}}]
```

```
[19]: # Circuit gate representation
for i in range(len(exe_ApplicationNames)):
    Gates_Representations = qsoa.representResults(restok_Executions[i])
```





As it can be seen in the results, the simulations coincides with what was predicted in theory. The state we have measured is $|00000\rangle$.

References

- [1] aQuantum, *QPath[®] Python SDK User Guide*. Available on QPath[®].